Neutrinoless double beta decay in four-neutrino models

Anna Kalliomäki [1] and Jukka Maalampi [2]

Department of Physics, Theoretical Physics Division

University of Helsinki, Finland

ABSTRACT

The most stringent constraint on the so-called effective electron neutrino mass from the present neutrinoless double beta decay experiments is $|M_{ee}| < 0.2$ eV, while the planned next generation experiment GENIUS is anticipated to reach a considerably more stringent limit $|M_{ee}| < 0.001$ eV. We investigate the constraints these bounds set on the neutrino masses and mixings of neutrinos in four-neutrino models where there exists a sterile neutrino along with the three ordinary neutrinos. We find that the GENIUS experiment would be sensitive to the electron neutrino masses down to the limit $m_{\nu_e} \lesssim 0.024$ eV in such a scenario.

[1]E-mail address: amkallio@pcu.helsinki.fi

[2]E-mail address: maalampi@pcu.helsinki.fi

There exist strong experimental indications that neutrinos have mass and that they mix. The results of the Super-Kamiokande experiment [1] on neutrinos produced in Earth's atmosphere by cosmic rays provide a convincing evidence of neutrino oscillations, and the results of the solar neutrino experiments [2] point to the same direction. Results from the laboratory experiment by the LSND collaboration [3] may also be an indication of neutrino oscillations but they still await a future confirmation, e.g. at the MiniBoone and I216 experiments [4], in particular so because measurements in KARMEN detector [5] exclude most of the region in the parameter space favoured by the LSND.

Neutrino oscillation probabilities are determined by squared mass differences $\delta m_{ji}^2 = m_j^2 - m_i^2$, where m_i is the mass of the massive neutrino state ν_i, and the elements of the neutrino mixing matrix U that connects the massive neutrino states ν_i and the flavour neutrino states ν_α through the relation $\nu_\alpha = \sum_i U_{\alpha i} \nu_i$. For explaining the solar and atmospheric neutrino data two mass-squared difference scales, $\delta m_{\text{atm}}^2 \simeq 10^{-3} - 10^{-2}$ eV^2 for atmospheric neutrinos and $\delta m_{\text{sun}}^2 \simeq 10^{-5}$ eV^2 or $\simeq 10^{-10}$ eV^2 for solar neutrinos, are needed, and the data can be described within a three-neutrino model. If the LSND result is not disregarded, a three-neutrino framework is not sufficient but one has to go beyond it and consider a model with four (or more) neutrinos as three different levels of mass-squared differences are needed, the third one being $\delta m_{\text{LSND}}^2 = 0.3 - 2$ eV^2. In addition to the three known neutrino flavours ν_e, ν_μ and ν_τ one has to incorporate at least one light neutrino ν_s that is sterile, i.e. it lacks Standard Model (SM) gauge interactions [6].

As was pointed out in ref. [7], the only four-neutrino mass patterns that are consistent with all oscillation data are those where there are two pairs of neutrinos with close masses, the "sun-pair" (mass states ν_0 and ν_1) and "atm-pair" (mass states ν_2 and ν_3) corresponding to the squared-mass differences δm_{atm}^2 and δm_{sun}^2, respectively, separated by a gap corresponding to δm_{LSND}^2 .

Another oscillation constraint follows from cosmology. Depending on the value of δm^2, a large active-neutrino mixing could bring sterile neutrinos into thermal equilibrium thereby increasing the effective number of light neutrinos [8], N_ν, which would affect the big bang nucleosynthesis (BBN). This leads to quite tight constraints on the active-sterile mixings [8]–[11].

In the case neutrinos are Majorana particles [12], which is quite likely from the theoret-

ical point of view, another useful source of experimental information on neutrino masses and mixings is provided by the neutrinoless double beta ($0\nu\beta\beta$) decay. The $0\nu\beta\beta$ decay is sensitive to the values of the neutrino masses themselves, in contrast with the oscillation data which depend only on the squared-mass differences. Also, it provides information on the Majorana CP-phases of neutrinos, to which the oscillation phenomena are practically insensitive. In the present work we shall study the constraints the $0\nu\beta\beta$ decay places on the neutrino masses m_i, the neutrino mixing matrix U and the relative CP-phases of neutrinos in four-neutrino models consisting of three active neutrinos ν_e, ν_μ and ν_τ and a sterile neutrino ν_s with the 2+2 mass pattern described above.

The quantity probed in the $0\nu\beta\beta$ decay experiments is the absolute value of the effective mass

$$M_{ee} = \sum_i m_i U_{ei}^2. \tag{1}$$

The most recent experimental upper bound for it from the Moscow-Heidelberg experiment is (at 90% C.L.) [13]

$$|M_{ee}| < 0.2\,\mathrm{eV}. \tag{2}$$

In turns out that this bound is in the case of four-neutrino models less restrictive than in the case of the tree-neutrino models [14]. A substantial strengthening of the present bound is, however, foreseen in the future. In the planned GENIUS experiment one expects to reach after the first year of operation the limit 0.01 eV and eventually the limit [15]

$$|M_{ee}| < 0.001\,\mathrm{eV} \quad \text{(GENIUS)}. \tag{3}$$

We will study the restrictions this bound would set on the parameters of four-neutrino models, in particular on the mass of the electron neutrino ν_e and the mixing of ν_e with ν_μ and ν_τ.

The constraint from the $0\nu\beta\beta$ decay is sensitive to the relative CP-phases η_i of the mass states ν_i. We will concentrate in the CP-conserving case where $\eta_i = \pm 1$. In this case one can write (1) in the form [16]

$$M_{ee} = \sum_i \eta_i m_i |U_{ei}|^2. \tag{4}$$

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The most general mixing matrix U that describes the connection between the flavour states $\nu_s, \nu_e, \nu_\mu, \nu_\tau$ and the Majorana mass eigenstates $\nu_0, \nu_1, \nu_2, \nu_3$ can be parametrized in terms of 6 rotation angles and 6 phases as follows (see, e.g., ref. [17]):

$$U = \begin{pmatrix} c_{01}c_{02}c_{03} & c_{02}c_{03}s_{01}^* & c_{03}s_{02}^* & s_{03}^* \\[2ex] \begin{aligned} &-c_{01}c_{02}s_{03}s_{13}^* \\ &-c_{01}c_{13}s_{02}s_{12}^* \\ &-c_{12}c_{13}s_{01} \end{aligned} & \begin{aligned} &-c_{02}s_{01}^*s_{03}s_{13}^* \\ &-c_{13}s_{01}^*s_{02}s_{12}^* \\ &+c_{01}c_{12}c_{13} \end{aligned} & \begin{aligned} &-s_{02}^*s_{03}s_{13}^* \\ &+c_{02}c_{13}s_{12}^* \end{aligned} & c_{03}s_{13}^* \\[3ex] \begin{aligned} &-c_{01}c_{02}c_{13}s_{03}s_{23}^* \\ &+c_{01}s_{02}s_{12}^*s_{13}s_{23}^* \\ &-c_{01}c_{12}c_{23}s_{02} \\ &+c_{12}s_{01}s_{13}s_{23}^* \\ &+c_{23}s_{01}s_{12} \end{aligned} & \begin{aligned} &-c_{02}c_{13}s_{01}^*s_{03}s_{23}^* \\ &+s_{01}^*s_{02}s_{12}^*s_{13}s_{23}^* \\ &-c_{12}c_{23}s_{01}^*s_{02} \\ &-c_{01}c_{12}s_{13}s_{23}^* \\ &-c_{01}c_{23}s_{12} \end{aligned} & \begin{aligned} &-c_{13}s_{02}^*s_{03}s_{23}^* \\ &-c_{02}s_{12}^*s_{13}s_{23}^* \\ &+c_{02}c_{12}c_{23} \end{aligned} & c_{03}c_{13}s_{23}^* \\[3ex] \begin{aligned} &-c_{01}c_{02}c_{13}c_{23}s_{03} \\ &+c_{01}c_{23}s_{02}s_{12}^*s_{13} \\ &+c_{01}c_{12}s_{02}s_{23} \\ &+c_{12}c_{23}s_{01}s_{13} \\ &-s_{01}s_{12}s_{23} \end{aligned} & \begin{aligned} &-c_{02}c_{13}c_{23}s_{01}^*s_{03} \\ &+c_{23}s_{01}^*s_{02}s_{12}^*s_{13} \\ &+c_{12}s_{01}^*s_{02}s_{23} \\ &-c_{01}c_{12}c_{23}s_{13} \\ &+c_{01}s_{12}s_{23} \end{aligned} & \begin{aligned} &-c_{13}c_{23}s_{02}^*s_{03} \\ &-c_{02}c_{23}s_{12}^*s_{13} \\ &-c_{02}c_{12}s_{23} \end{aligned} & c_{03}c_{13}c_{23} \end{pmatrix}, \tag{5}$$

where $c_{jk} \equiv \cos\theta_{jk}$ and $s_{jk} \equiv \sin\theta_{jk}e^{i\delta_{jk}}$. For our discussion relevant is the second row which gives the composition of the electron neutrino in terms of the mass eigenstates neutrinos. It depends on several mixing parameters but fortunately simplifies considerably when the existing constraints from laboratory experiments and cosmology are taken into account.

Assuming the 2+2 mass hierarchy among neutrino masses, indicating $\delta m_{02}^2 \simeq \delta m_{03}^2 \simeq \delta m_{12}^2 \simeq \delta m_{13}^2 \simeq \delta m_{LSND}^2$, the Bugey short baseline experiment [18], measuring the probability $P(\nu_e \to \nu_e) \simeq 1 - A^{ee}\sin^2(L\delta m_{LSND}^2/4E)$, yields the bound [17]

$$A^{ee} = 4(|U_{e2}|^2 + |U_{e3}|^2)(1 - |U_{e2}|^2 - |U_{e3}|^2) < 0.06, \tag{6}$$

or

$$|c_{02}c_{13}s_{12}^* - s_{02}^*s_{03}s_{13}^*|^2 + |c_{03}s_{13}^*|^2 < 0.016. \tag{7}$$

The angles θ_{02} and θ_{03} are further constrained by the big bang nucleosynthesis (BBN) as large mixings would, as mentioned above, increase the number of the effective degrees of freedom N_ν by bringing the sterile neutrino into thermal equilibrium. According to a recent analysis [11], the two-flavour $\nu_{\tau,\mu} \leftrightarrow \nu_s$ mixings should obey the constraint (valid for $|\delta m^2| < 2.5 \cdot 10^3 \ \text{eV}^2$)

$$|\delta m^2| \sin^2 \theta < 7 \cdot 10^{-5} \ \text{eV}^2 \tag{8}$$

in order not to lead to a conflict with the BBN limit $N_\nu \lesssim 3.2$ [19]. Since now $\delta m^2 = \delta m^2_{\text{LSND}} = 0.3 - 2 \ \text{eV}^2$, the mixing angle θ should be $\lesssim 10^{-2}$, and it is therefore conceivable to assume $s_{02}, s_{03} \ll 1$. With this approximation one derives from (7) the bounds

$$\begin{aligned} b &\equiv \sin^2 \theta_{12} + \sin^2 \theta_{13} \lesssim 0.016, \\ |b'| &\equiv |\sin^2 \theta_{12} - \sin^2 \theta_{13}| \lesssim 0.016, \end{aligned} \tag{9}$$

and the following approximative composition of the electron neutrino in terms of the mass eigenstates:

$$\nu_e = -\sin \theta_{01} e^{i\delta_{01}} \nu_0 - \cos \theta_{01} \nu_1 + \sin \theta_{12} e^{i\delta_{12}} \nu_2 + \sin \theta_{13} e^{i\delta_{13}} \nu_3. \tag{10}$$

Let us note that the cosmological argument used above remains still somewhat controversial. Some analysis allow for the number of the relativistic degrees of freedom, instead of the bound $N_\nu \lesssim 3.2$ quoted above, values close to four [20], in which case BBN would not place any constraint on the angles s_{02}, s_{03}. The difference is due to conflicting results on the primordial deuterium abundance. Furthermore, the BBN constraint depends on the lepton asymmetry in the early universe, which might considerably weaken the bounds [21] (in the case the predominantly sterile mass eigenstate is lighter than its active mixing partner). Anyhow, we will assume in the following that the sterile neutrino mixes considerably only with the electron neutrino and consequently that the approximations (9) and (10) are valid.

In this approximation the solar neutrino oscillation is described entirely in terms of the angle θ_{01} associated with the $\nu_e \leftrightarrow \nu_s$ mixing. According to an overall analysis of all solar neutrino data, only the small angle matter solution (SAMSW) with $2 \cdot 10^{-3} \lesssim \sin^2 2\theta_{01} \lesssim 10^{-2}$ leads to a reasonable fit in the active-sterile mixing case [22].

There are two mass patterns consistent with the oscillation data, differing in as to which one of the pairs, the sun-pair or the atm-pair, is lighter. We will call Model A the pattern where the sun-pair is the light one and the atm-pair is the heavy one, and the case where the mass order is the opposite will be called Model B. The mass scale of the lighter neutrino pair is denoted by m, and the mass gap between the pairs is denoted by Δm, so that the mass scale of the heavier pair is $m + \Delta m$. We will neglect the mass differencies inside the two pairs by assuming they to be small as compared with the mass gap Δm.

Studies of the anisotropies of the cosmic microwave background radiation and the large scale structure of the universe provide information on the sum of neutrino masses, $\sum_i m_{\nu_i} \simeq 4m + 2\Delta m$. The contribution of neutrinos on the dark matter content of the Universe is in units of the critical density given by $\Omega_\nu = h^{-2} \sum_i m_{\nu_i}/94$ eV, where h is the dimensionless Hubble constant ($H_0 = 100\, h$ km s^{-1} Mpc^{-1}). In the previously popular hot+cold dark matter scenario [23] the neutrino contribution to the energy density of the universe is assumed to be $\Omega_\nu \simeq 0.2$, but the more recent observations, indicating the existence of a large cosmological constant, imply that this should be taken as a conservative upper limit only [24]. With the recent measured value of the Hubble constant [25], $h \simeq 0.71$, this leads to the upper limit $\sum_i m_{\nu_i} \lesssim 10$ eV. Recently it has been claimed that this limit improves to

$$\sum_i m_{\nu_i} \lesssim 5.5 \ \text{eV} \tag{11}$$

when the information from the Lyα forest in quasar spectra is taken into account [26].

The range of the squared mass difference, $\delta m^2_{\text{LSND}} = 0.3 - 2$ eV^2, indicated by the LSND data and allowed by the Bugey and the other short baseline experiments yields the following upper and lower bounds for the mass difference Δm as a function of the mass m:

$$-m + \sqrt{m^2 + 0.3 \ \text{eV}^2} \ \lesssim \ \Delta m \ \lesssim \ -m + \sqrt{m^2 + 2 \ \text{eV}^2}. \tag{12}$$

Combining this with the cosmological bound (11) leads to the following bounds on m and

Δm:

$$m \lesssim 1.3 \text{ eV},$$

$$\Delta m \simeq \frac{\delta m_{\mathrm{LSND}}}{\sum_i m_i/2} \geq 0.1 \text{ eV}. \tag{13}$$

In the extreme case of $m = 1.3\,\mathrm{eV}$ and $\Delta m = 0.1\,\mathrm{eV}$ the mass spectrum is quite degenerate with $m_0 \simeq m_1 \simeq 1.3$ eV and $m_2 \simeq m_3 \simeq 1.4$ eV (Model A) or $m_0 \simeq m_1 \simeq 1.4$ eV and $m_2 \simeq m_3 \simeq 1.3$ eV (Model B). At the other extreme of $m = 0$, i.e. when the lighter neutrino pair is massless, the heavier neutrino pair would have its mass in the range 0.5 eV to 1.4 eV.

Let us proceed to examine the consequences of the constraints (2) and (3) on the neutrino masses and mixing angles θ_{01}, θ_{12} and θ_{13} for different CP-phase patterns $(\eta_0, \eta_1, \eta_2, \eta_3)$ in the two models A and B.

Model A. In this model the neutrino pair (ν_0, ν_1) responsible for the solar anomaly is lighter than the pair (ν_2, ν_3) responsible for the atmospheric neutrino anomaly, i.e. the mass spectrum is of the form

$$m_0 \simeq m_1 = m$$

$$m_2 \simeq m_3 = m + \Delta m. \tag{14}$$

The $0\nu\beta\beta$ condition takes in this case the form

$$|m(\eta_0 \sin^2 \theta_{01} + \eta_1 \cos^2 \theta_{01}) + (m + \Delta m)(\eta_2 \sin^2 \theta_{12} + \eta_3 \sin^2 \theta_{13})| < a, \tag{15}$$

where for the present experimental bound $a = 0.2$ eV and for the anticipated future bound of the GENIUS experiment $a = 0.001$ eV. In order to find out the consequences of this condition one should examine separately the following different patterns of the relative CP-phases $\eta = (\eta_0, \eta_1, \eta_2, \eta_3)$:

$$\begin{aligned} \mathrm{I, II} \quad & \eta = (1, 1, \pm 1, \pm 1), \\ \mathrm{III, IV} \quad & \eta = (1, 1, \pm 1, \mp 1), \\ \mathrm{V, VI} \quad & \eta = (1, -1, \pm 1, \pm 1), \\ \mathrm{VII, VIII} \quad & \eta = (1, -1, \pm 1, \mp 1). \end{aligned} \tag{16}$$

In the first four cases (I-IV), where $\eta_0 = \eta_1$, the $0\nu\beta\beta$ condition is not affected by the solar neutrino mixing angle θ_{01}.

In Cases I and II the $0\nu\beta\beta$ condition (15) becomes

$$|m \pm (m + \Delta m)b| < a \tag{17}$$

where the plus-sign corresponds to Case I and the minus-sign to Case II, and $b = \sin^2 \theta_{12} + \sin^2 \theta_{13}$, as defined in eq. (9).

Case I. The LSND and other short baseline experiments constrain the value of the mass gap Δm between the two neutrino pairs within the range given in eq. (12). Combining this with the cosmological lower bound (13) yields for Δm the allowed range from 0.1 eV to 1.4 eV. One then concludes from (17) that with the present experimental sensitivity the $0\nu\beta\beta$ result does not set any constraint on the mixing angles θ_{12}, θ_{13} as $b\Delta m \ll a$. The experimental constraint implies just an upper limit for the mass scale m, $m \lesssim (a - b\Delta m)/(1 + b) \simeq a = 0.2$ eV.

The situation would be different in the case of the anticipated GENIUS bound $a = 0.001$ eV. The condition (17) to be satisfied one must now have $b \leq a/\sqrt{0.3}$ eV $\simeq 2a/\text{eV} = 0.002$, which would mean tightening of the present bound (9) from the Bugey reactor experiment by one order of magnitude. Also the allowed values of m are relatively small, $m \lesssim a = 0.001$ eV.

Case II. The Case II does not differ from Case I in any essential way as far as the present $0\nu\beta\beta$ bound is concerned. With the GENIUS bound the upper limit of the mixing angles θ_{12}, θ_{13} is as stringest $b \lesssim a/\sqrt{0.3}$ eV $\simeq 2a/\text{eV} = 0.002$ obtained for $m = 0$. The situation is different compared with Case I in that now the condition (17) imposes also a non-zero lower limit on b when $m \gtrsim a$, otherwise there would not be sufficient cancellations among the varios terms to make the condition (15) to be valid. The quantity b would be constrained into the range $(m-a)/\sqrt{m^2 + 2 \text{ eV}^2} \lesssim b \lesssim (m+a)/\sqrt{m^2 + 0.3 \text{ eV}^2}$, where the upper limit is overtaken by the upper limit $b \lesssim 0.016$ from the short baseline experiments, when $m \gtrsim 0.008$ eV. The sensitivity range of the GENIUS experiment in the (m, b) parameter plane is presented in Fig. 1 (solid line). As can be read from the figure, the GENIUS experiment would in this case be sensitive to the values of m down to 0.024 eV.
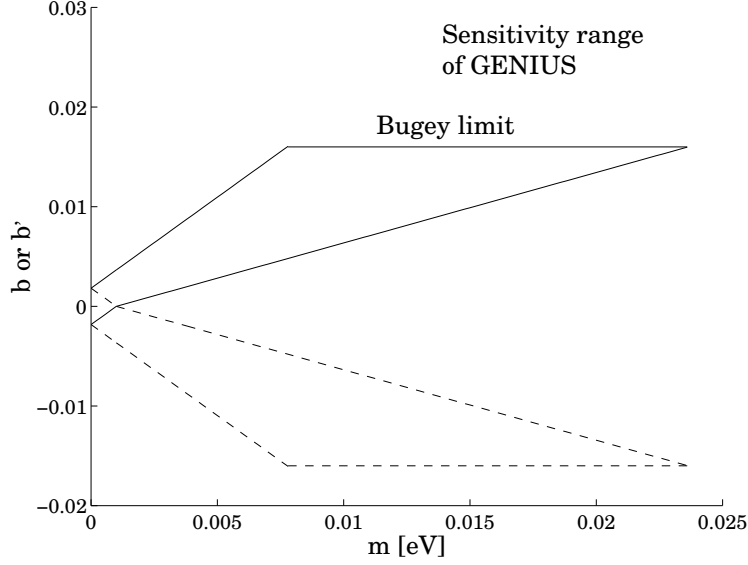
Figure 1: The sensitivity range reached by the planned GENIUS experiment in the mass parameter m and the mixing angle variables $b = \sin^2\theta_{12} + \sin^2\theta_{13}$ and $b' = \sin^2\theta_{12} - \sin^2\theta_{13}$ in Model A. The upper and/or lower bounds of ± 0.016 for b and b' are obtained from Bugey experiment [18]. The area bounded by the dashed line illustrates Cases I (b as vertical axis) and III (b' as vertical axis) (see eq.(16)) whereas the area bounded by solid line illustrates the sensitivity range in Cases II (b) and IV (b'). Cases V-VIII are related to Cases I-IV in a way described in the text.

Cases III and IV. In Cases III and IV the $0\nu\beta\beta$ condition (15) becomes

$$|m \pm (m + \Delta m)b'| < a, \tag{18}$$

where the plus-sign corresponds to Case III and the minus-sign to Case IV, and $b' = \sin^2\theta_{12} - \sin^2\theta_{13}$, as defined in (9). Just like in Cases I and II, the present $0\nu\beta\beta$ bound does not probe the mixing angles θ_{12} and θ_{13} but sets a constraint merely on the mass m. Again, the GENIUS experiment would be sensitive to the values of these angles. In Case III the condition (18) restricts the allowed values of $b' = \sin^2\theta_{12} - \sin^2\theta_{13}$, whose value is by other constraints limited to $-0.016 \lesssim b' \lesssim 0.016$, so that for example for $m = 0$ only the values $|b'| \leq a/\sqrt{0.3}$ eV $\simeq 2a/\text{eV} = 0.002$ are allowed. The condition (18) is never satisfied for the mass values $m \gtrsim 0.024$ eV. This would be the sensitivity limit of

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the GENIUS on the electron neutrino mass also in this case. The allowed values of b' are negative, $(-m - a)/\sqrt{m^2 + 0.3 \text{ eV}^2} \lesssim b' \lesssim (-m + a)/\sqrt{m^2 + 2 \text{ eV}^2}$, except that for $m < a$ small positive values are also possible. The GENIUS sensitivity range for this case is presented in Fig. 1 with the dashed line. Case IV is a mirror image of Case III so that again for $m = 0$ all values $|b'| \lesssim a/\sqrt{0.3}$ eV are possible, but now b' can have mainly positive values: $(m - a)\sqrt{m^2 + 2 \text{ eV}^2} \lesssim b' \lesssim (m + a)/\sqrt{m^2 + 0.3 \text{ eV}^2}$ (the solid line in Fig. 1). Note that this is almost the same region that was obtained in Case II for the quantity b. The limit that the GENIUS experiment could reach for the mass m is again 0.024 eV.

Cases V and VI. In these cases the $0\nu\beta\beta$ condition depends also on the solar mixing angle θ_{01}:

$$|m(\sin^2 \theta_{01} - \cos^2 \theta_{01}) \pm (m + \Delta m)b| < a, \tag{19}$$

where the plus-sign corresponds to Case V and the minus-sign to Case VI. If only the small mixing angle (SMA) MSW solution is being considered, one can use the approximative value $\sin^2 \theta_{01} - \cos^2 \theta_{01} \simeq \pm 1$, where the sign depends on whether θ_{01} is close to 0 or $\pi/2$. If one assumes that angle $\theta_{01} \simeq 0$, condition (19) becomes

$$| - m \pm (m + \Delta m)b| \lesssim a. \tag{20}$$

As is apparent from eqs. (17) and (20), the $0\nu\beta\beta$ constraints in Cases II and V are in practical terms the same, and no separate treatment is needed for Case V. The same applies to Cases I and VI. On the other hand, if the angle θ_{01} is close to $\pi/2$, condition (20) must be replaced with

$$|m \pm (m + \Delta m)b| \lesssim a. \tag{21}$$

and clearly Cases I and V as well as Cases II and VI correspond to each other as far as the $0\nu\beta\beta$ constraint is concerned.

It is noteworthy that the values of θ_{01} close to 0 and close to $\pi/2$ correspond here two physically distinct situations while in oscillation experiments, where transition probabilities depend on θ_{01} through $\sin^2 2\theta_{01}$, do not make any difference between the ranges $0 \lesssim \theta_{01} \lesssim \pi/4$ and $\pi/4 \lesssim \theta_{01} \lesssim \pi/2$.

Cases VII and VIII. These cases are practically identical to Cases III and IV. If $\theta_{01} \simeq 0$, the $0\nu\beta\beta$ condition (15) takes the form

$$| - m \pm (m + \Delta m)b'| < a, \tag{22}$$

where the plus-sign corresponds to Case VII and minus-sign to Case VIII. Comparison between eqs. (18) and (22) shows that in Cases IV and VII as well as in Cases III and VIII the $0\nu\beta\beta$ condition is practically the same. If $\theta_{01} \simeq \pi/2$, the same applies to Cases III, VII and IV, VIII.

Model B. In this model the mass spectrum is of the form

$$m_0 \simeq m_1 = m + \Delta m$$
$$m_2 \simeq m_3 = m. \tag{23}$$

Again we assume that mixing of the neutrino pair (ν_0, ν_1) is mainly responsible for the solar neutrino anomaly and mixing of the pair (ν_2, ν_3) for the atmospheric neutrino anomaly. The $0\nu\beta\beta$ condition becomes now

$$|(m + \Delta m)(\eta_0 \sin^2 \theta_{01} + \eta_1 \cos^2 \theta_{01}) + m(\eta_2 \sin^2 \theta_{12} + \eta_3 \sin^2 \theta_{13})| < a. \tag{24}$$

If LSND results are taken into account and one assumes that $0.3\,\mathrm{eV}^2 \lesssim m_1^2 - m_3^2 \lesssim 2\,\mathrm{eV}^2$, which yields $m_1 \gtrsim 0.5\,\mathrm{eV}$, it is easy to see that eq. (24) is not realized for any value of m [27]. This is because the absolute values of b and b', defined in eq. (9), are much less than $|\eta_0 \sin^2 \theta_{01} + \eta_1 \cos^2 \theta_{01}| \simeq 1$, so that eq. (24) can be written approximately as

$$|m + \Delta m| \lesssim a, \tag{25}$$

which is untrue because $|m + \Delta m| = m_1$ and even the present value of a is less than the lower limit $0.5\,\mathrm{eV}$ of m_1. If we, on the other hand, disregard the LNSD results and let Δm be as low as $0.1\,\mathrm{eV}$, with present value of a we get some restrictions on the parameter values from eq. (24). Eight different η combinations (16) are in principle possible also in this case but because (as was already mentioned) the term proportional to b or b' in eq. (24) is much smaller than the term proportional to angle θ_{01}, it can safely be neglected. So, the approximative equation (25) is valid in every case, not depending on the choices

of η_0 and η_1 or whether θ_{01} is $\simeq 0$ or $\pi/2$. The allowed values for m are in the range $m \lesssim a - \Delta m$.

We can summarize our results as follows. In the model A, i.e. when the neutrino pair (with mass m) responsible on the solar neutrino deficit is lighter than the neutrino pair (with mass $m + \Delta m$) responsible on the atmospheric neutrino anomaly, the $0\nu\beta\beta$ constraint set by the future GENIUS experiment, $M_{ee} \lesssim 0.001$ eV, induces an upper limit for mass m as a function of the quantity $b = \sin^2 \theta_{12} + \sin^2 \theta_{13}$ or the quantity $b' = \sin^2 \theta_{12} - \sin^2 \theta_{13}$ depending on the pattern of the relative CP numbers of the neutrinos. There exists eight possible patterns of the relative CP numbers, Cases I to VIII, defined in eq. (16). In Case I, where all relative CP numbers are equal, and no cancellations between the contributions of different neutrinos hence occurs, the upper bound for m is the stringest, $m \lesssim 0.001$ eV. For all the others Cases the absolute mass bound is about an order of magnitude less stringent, $m \lesssim 0.024$ eV. In Case I the value of b must be small, $b \lesssim a$. In the other Cases the restriction on b or b' depend on the value of m (see Fig. 1), and both a lower and an upper limit are obtained. For $m \lesssim 0.08$ eV the foreseen upper limit for b or $|b'|$ is more stringent that the limit of the Bugey experiment. There is a crucial difference between the different relative CP number patterns as comes to the ensuing limitations on the mixing angles θ_{12} and θ_{13}, as a bound on b implies a bound on the both angles whereas a bound on b' implies only a bound on the difference of the angles, not on each of the angles separately. Also, the smaller is the mass m, the smaller or the more degenerate are the angles θ_{12} and θ_{13}, depending on whether the obtained bound is on b or b', respectively.

The Model B, i.e. when the neutrino pair responsible on the solar neutrino deficit is heavier than the neutrino pair responsible on the atmospheric neutrino anomaly, is in practical terms ruled out already by the present limit of $M_{ee} \lesssim 0.2$ eV.

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